Exercise 298

Rewrite the following expressions in terms of exponentials and simplify.

a. $2\cosh(\ln x)$

b. $\cosh 4x + \sinh 4x$

c. $\cosh 2x - \sinh 2x$

d. $\ln(\cosh x + \sinh x) + \ln(\cosh x - \sinh x)$

Solution

Use the definitions of hyperbolic sine and hyperbolic cosine to simplify these expressions.

$$\cosh x = \frac{e^x + e^{-x}}{2} \qquad \sinh x = \frac{e^x - e^{-x}}{2}$$

Part (a)

Note that e^x and $\ln x$ are inverse functions and that exponents in a logarithm argument can be brought down in front and vice-versa.

$$\begin{aligned} 2\cosh(\ln x) &= 2\left(\frac{e^{\ln x} + e^{-\ln x}}{2}\right) \\ &= e^{\ln x} + e^{-\ln x} \\ &= e^{\ln x} + e^{\ln x^{-1}} \\ &= x + x^{-1} \\ &= x + \frac{1}{x} \end{aligned}$$

Part (b)

$$\begin{aligned} \cosh 4x + \sinh 4x &= \left(\frac{e^{4x} + e^{-4x}}{2}\right) + \left(\frac{e^{4x} - e^{-4x}}{2}\right) \\ &= \frac{1}{2}e^{4x} + \frac{1}{2}e^{-4x} + \frac{1}{2}e^{4x} - \frac{1}{2}e^{-4x} \\ &= e^{4x} \end{aligned}$$

Part (c)

$$\begin{aligned} \cosh 2x - \sinh 2x &= \left(\frac{e^{2x} + e^{-2x}}{2}\right) - \left(\frac{e^{2x} - e^{-2x}}{2}\right) \\ &= \frac{1}{2}e^{2x} + \frac{1}{2}e^{-2x} - \frac{1}{2}e^{2x} + \frac{1}{2}e^{-2x} \\ &= e^{-2x} \end{aligned}$$

Part (d)

$$\begin{aligned}
\ln(\cosh x + \sinh x) + \ln(\cosh x - \sinh x) &= \ln\left[\left(\frac{e^x + e^{-x}}{2}\right) + \left(\frac{e^x - e^{-x}}{2}\right)\right] + \ln\left[\left(\frac{e^x + e^{-x}}{2}\right) - \left(\frac{e^x - e^{-x}}{2}\right)\right] \\
&= \ln\left(\frac{1}{2}e^x + \frac{1}{2}e^{-x} + \frac{1}{2}e^x - \frac{1}{2}e^{-x}\right) + \ln\left(\frac{1}{2}e^x + \frac{1}{2}e^{-x} - \frac{1}{2}e^x + \frac{1}{2}e^{-x}\right) \\
&= \ln e^x + \ln e^{-x} \\
&= (x)\ln e + (-x)\ln e \\
&= x\ln e - x\ln e \\
&= 0
\end{aligned}$$